CID No: ________________________

IMPERIAL COLLEGE LONDON

**Design Engineering MEng EXAMINATIONS 2024**

For Internal Students of the Imperial College of Science, Technology and Medicine
*This paper is also taken for the relevant examination for the Associateship or Diploma*

**DESE71004 – Design of Visual Systems**

Date: 30 April 2024
Time: 10.00am – 11.30am
Duration: One hour thirty minutes

*This paper contains EIGHT questions.*
*Full marks of the paper is 100 out of 100.*
*Attempt ALL questions.*

*The numbers of marks shown by each question are for your guidance only; they indicate how the examiners intend to distribute the marks for this paper.*

*Students are allowed to bring to the examination one double sided A4 sheet of handwritten information of their own choosing.*

1) The intensity values $r$ of an image are continuous and have the probability distribution function (PDF) of $p_r(r)$, where:

$p_r(r) = 2r/\,(L-1)^2$ for $0 \le r \le L-1,$

$p_r(r) = 0$ for all other values of $r$.

The image is histogram-equalized with the intensity transformation function $T$, such that the new image intensity values is given by $s = T(r)$.

Prove that

$$s = T(r) = \frac{r^2}{(L-1)}.$$

[12]

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**Solution to Q1**

This question tests student's understanding of the principle of histogram equalization. The test is on the following three key ideas: 1) the probability distribution function (PDF) is the normalised version of the intensity histogram of an image; 2) the cumulative distribution function (CDF) is the intensity transformation required to achieve histogram equalization; 3) the cumulative distribution function is the integral of the probability distribution function (PDF).

By definition, $s = T(r) = 0$ for all values $r$. outside the range of $[0, L-1]$.

Therefore, the histogram equalization transformation $T(r)$ for the intensity range [0, L-1] is the integral of the PDF function over the range of $[0, r]$, and scaled up by $(L-1)$.

Hence:

$$s = T(r) = (L-1)\int_0^r p_r(x)\,dx = (L-1)\int_0^r 2x\,/(L-1)^2 dx$$

$$= \frac{2}{(L-1)}\int_0^r x\,dx = \frac{r^2}{(L-1)}.$$

[12]

COMMENTS:

Most students got this one perfectly. Some did not provide full detail explanation beyond a simple equation and the results of integration, and lost a few marks.

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2) The image $f$ and the filter kernel $w$ are given in *Figure Q2*.

a) Compute the output image $g$, which is the filtered version of $f$ with the kernel $w$ using convolution. That is:

$$g = w * f.$$

State any assumptions used.

[8]

b) What is the output image if filtering is done using correlation instead of convolution?

[4]

$$w = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \qquad f = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

*Figure Q2*

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**Solution to Q2**



This question tests student's understanding of filtering of an image with convolution, and that correlation and convolution are the same if the filter kernel is symmetrical (as is the case here).

a) Assume that we zero-padded the input image with one extra row and column around the border, and that the final image is trimmed back to the original size.

$$g = \begin{bmatrix} 0 & 1 & 2 & 1 & 0 \\ 0 & 3 & 6 & 3 & 0 \\ 0 & 3 & 6 & 3 & 0 \\ 0 & 1 & 2 & 1 & 0 \end{bmatrix}$$

[8]

*b)* The correlation result will be the same as convolution because the kernel is symmetrical.

[4]

COMMENTS:

Most students got this right, demonstrating that they understood the 2D convolution operation.

3) The R, G, and B component images of an RGB image have the horizontal intensity profiles shown in *Figure Q3*. What colour would a person see in the middle column of this image?

[6]

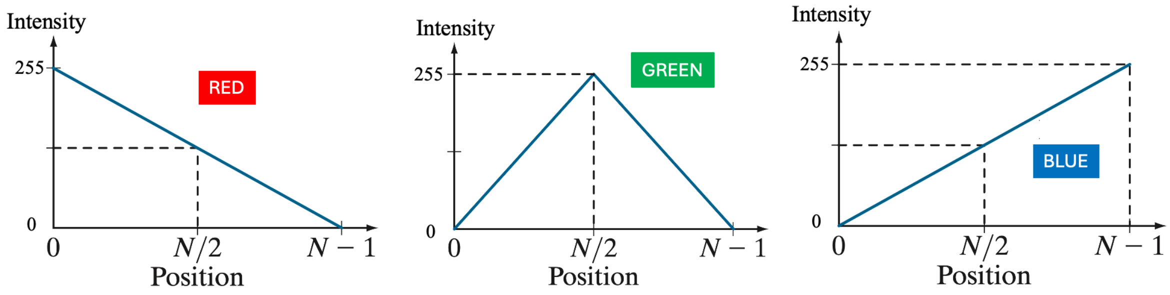


*Figure Q3*

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**Solution to Q3**

This question tests student's understanding how R, G and B colours are mixed to form grayscale and determine the intensity. At the centre point (the location of interest), the three changes are:

$$\frac{1}{2}R + \frac{1}{2}B + G = \frac{1}{2}(R + G + B) + \frac{1}{2}G = \text{midgray} + \frac{1}{2}G.$$

Therefore, the colour seen by a person is pure green with boosted intensity due to the additive gray component.

[6]

COMMENTS:

Most students got the correct answer.

4) In *Figure Q4* below, (a) is a binary image with a white object in a black background. A morphological operator is performed on this image with the structuring element shown in (b) to obtain the output (c). Note that the output is drawn with the original foreground object shown in dotted line and the resultant object shown in grey.

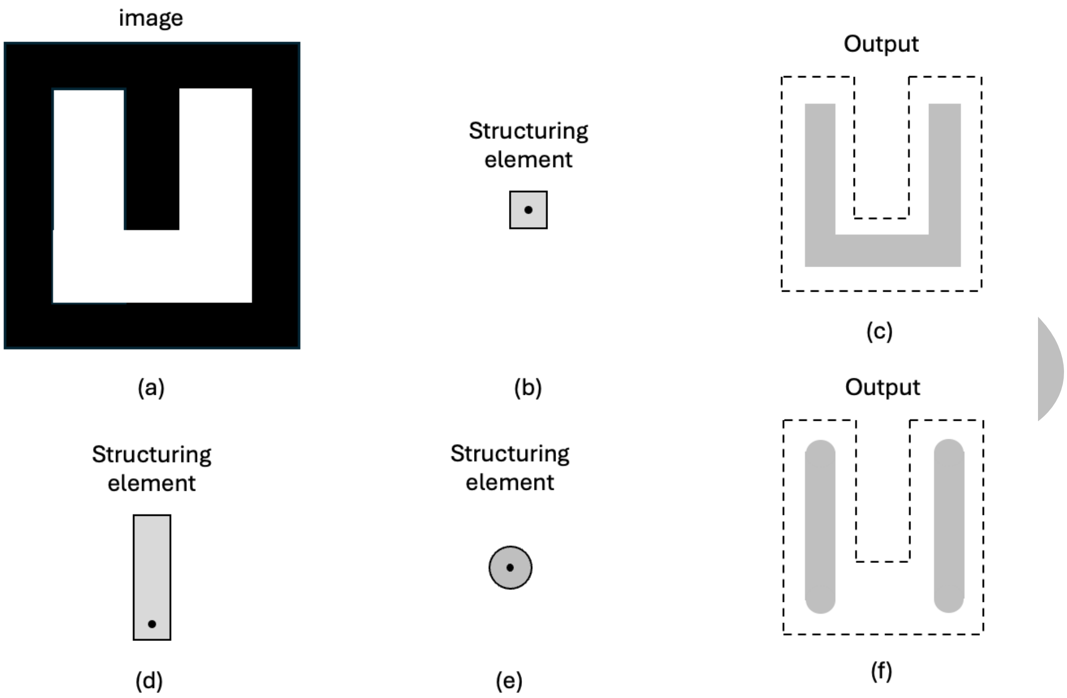



*Figure Q4*

a) What morphological operation of image (a) produces the output shown in (c)? [4]

b) Sketch the output when the image (a) is eroded with the structuring element shown in (d)? [5]

c) Sketch the output when the image (a) is dilated with the structuring element shown in (e)? [5]

d) What structuring elements and morphological operations produces the output shown in (f)? [6]

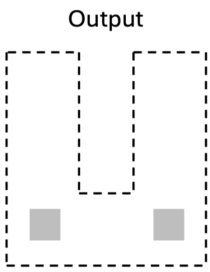
## Solution to Q4

This question tests student's understanding of morphological operations.

a) The operation is erosion.

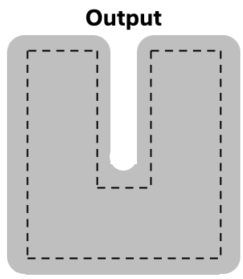
[4]

b) The eroded output image is:




[5]

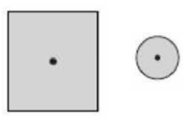
c) The dilated output image is:




[5]

d) To generate this image, first erode the object down to two vertical lines using the rectangular structuring element. This element must be slightly taller than the center section of the "U" object to leave just two vertical lines on the side elements.

Then dilate the line with a circular structuring element with diameter the same as the require width of the resulting objects.



[6]

COMMENTS:

This is the more challenging question so far. Most got b) slightly wrong and d) proved to be hardest of all. A) and c) were OK.

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5) *Figure Q5* shows a grayscale image with an intensity range of $[0, 255]$. It is known that the mean intensities of the objects and background are 170 and 60 respectively, and that the image has been corrupted by Gaussian noise with zero mean and a standard deviation of intensity level of 10.

Propose with justifications a sequence of steps to extract the objects as a binary image with the objects in the foreground.

[12]

*Figure Q5*

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**Solution to Q5**

This question tests student's ability to apply techniques they have learned to perform effective segmentation. There is no unique answer. However, a good answer would be:

1) Step 1: Filter the image with a lowpass Gaussian filter to reduce the noise. The $\sigma$ value of the Gaussian kernel must be narrow enough to avoid overly smooth the boundary between the object and the background.

2) Step 2: Apply Otsu's method to determine a good value as the threshold. Since the difference between the means intensity of the foreground and background is 110, and that the standard deviation of the Gaussian noise is only 10, the histogram of intensity is clearly bimodal. Therefore, Olsu's method should work well.

3) Use the threshold value to make every pixel equal or above the threshold to be ‘1’ or foreground, and the rest ‘0’.

[12]

COMMENTS:

Most did okay with this question. Some gave details relating to scale of the features (e.g. size of low pass filter kernel), but those who just gave a “handwaving” answers lost a few marks. Some stated using thresholding method without justifications and also lost marks.

6) *Figure Q6* shows two pattern classes $A$ and $B$, with two dimensions of features $x_1$ and $x_2$. All instances of class $A$, shown as red stars, plot on the $x_1$ and $x_2$ feature space, are inside the circle $C_1$ with radius $r_1$. All instances of class $B$, shown as blue squares are outside the circle $C_2$ with radius $r_2$. It is also known that $r_2 > r_1$, and that the centre of $C_2$ is inside $C_1$.

Specify in the form of a diagram and with written justification, the structure of a neural network with the minimum number of layers and nodes needed to classify the patterns of these two classes.

[12]

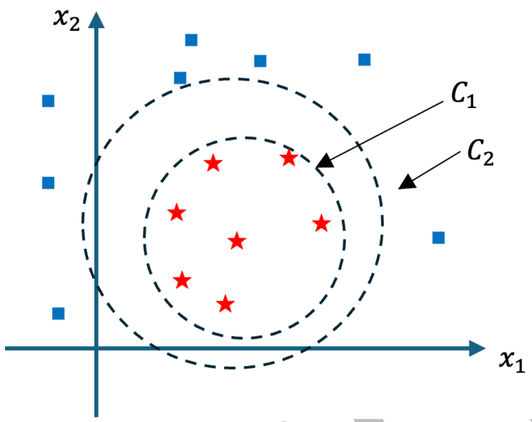


*Figure Q6*

**Solution to Q6**

This question tests student's understanding why neural network in the form of multilayer perceptron are effective classifiers.

Draw four lines as shown below enclosing the inner circle in such a way that all four intersections of the lines are between the two circles.

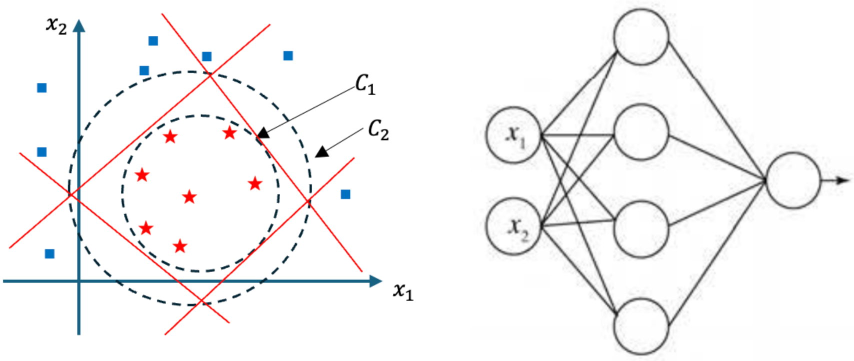


*Figure Q6*

Since a perceptron is a universal linear classifier that separate the two-dimensional feature space into two classes, we can build a neural network with only one hidden layer with four perceptrons (or neurons). The output layer perceptron simply combines all four categories such that on those inside the four linear boundaries belong to class A.

Note that the solution MUST specify the intersections of the four lines lie between the two circles. Otherwise, the neural network will misclassify some of the samples.

[12]

COMMENDS:

Most students failed to provide the left hand solution showing how the space was divided into four separate regions as, which provides justification for the four hidden layer neurons.

7) Your classmate asked you to explain, in 2 to 3 minutes, how information from a visual scene gets to the brain.

Compose, in approximately 250-300 words, the pathway through which light reaches the brain in a human. DO NOT include what the brain does with such information.

[12]

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**Solution to Q7**

This question tests student's knowledge on how visual information in the physical world is transformed into neural signals for the brain.

Here is a very full description that is beyond what students are expected to produce. The key words are shown in **bold** and indicate the terms that are expected to see in the description.

- **Visible light** in the form of **photons**, **reflected** or **transmitted**, enters the eye through the **transparent cornea**.
- The **lens**, a crystalline structure, **converges** and **focuses** light onto retina, which is full of **photoreceptors**.
- The photoreceptors in the retina, called **rods** and **cones**, containing **light-sensitive pigments** *(called rhodopsin). (Photons causes these cells emit transducin and closes a chemical ion channel, which triggers electrical signals to be produced.)*
- The generated **electrical signals** (propagate through bipolar and horizontal cells) converging onto retinal **ganglion** cells.
- **Axons** from retinal ganglion cells together form the **optic nerve**, which carries the visual information towards the brain through the blind spot.
- **Rods** are sensitive to **low light** and sensitive to a **wide colour spectrum**,
- **Three types of cones** are sensitive to three separate **regions** of the **colour spectrum** roughly around the red, green, and blue colours.
- The retina has a small **fovea** region, full of **cones**, is responsible for **colour and detail** vision.
- The rest of the retina is full of **rods** and is responsible for **peripheral** vison.
- There is also a **blind spot** where the **optic nerve** goes through to the brain.
- **Axons** from retinal ganglion cells together form the **optic nerve**, which carries the visual information towards the brain through the blind spot.
- The optic nerve reaches the **optic chiasm** and the optic nerve fibres from the **right and left** part of each eye **cross over** in such a way that the right vision is processed by the left brain and vice versa.
- The optic nerve fibres terminate in the **lateral geniculate nucleus** (LGN) of the **thalamus**, which relays the visual information to the primary **visual cortex** located in the occipital lobe of the cerebral cortex.

(280 words)
[12]

COMMENTS: Most students did reasonably well on this question. Some were comprehensive and some were sketchy.

8) You are designing a prototype virtual reality headset device. An OLED display panel is positioned 40 mm behind a lens which will act to produce a virtual image of the display panel 800 mm in front of the viewer. That is, light from one of the pixels of the display will be imaged to appear to originate 800 mm away as shown in *Figure Q8.*

(i) Calculate the required focal length of the lens.

[4]

(ii) Initially assuming the thickness of the lens can be ignored, and the radii of curvature on both lens faces is the same – calculate the radius of curvature for the lens with a refractive index of 1.69.

[5]

(iii) For spherical lens faces, what is the minimum thickness of the lens if it has a diameter of 35 mm?

[5]

**Lens Formula:** $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

**Lens Maker Formula:** $\frac{1}{f} = (n-1)\left[\frac{1}{R_1} + \frac{1}{R_2} - \frac{(n-1)d}{nR_1R_2}\right]$

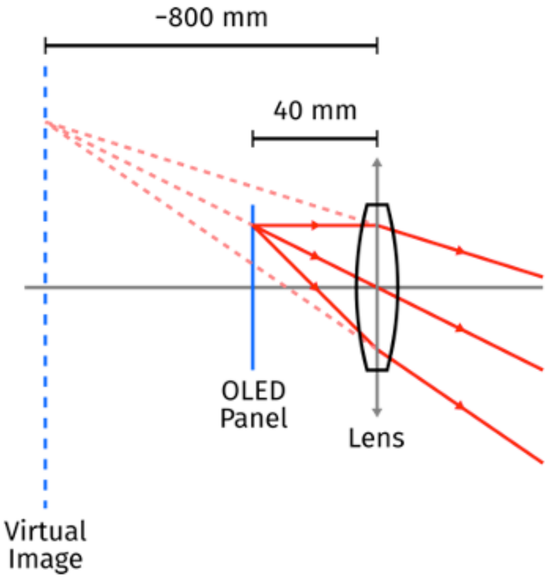


*Figure Q8*

## Solution to Q8

This question tests student's knowledge on optics relating to visual systems.

i) Lens formula, $\frac{1}{f} = \frac{1}{i} + \frac{1}{o}$, here: $\frac{1}{f} = \frac{1}{-800\text{ mm}} + \frac{1}{40\text{ mm}}$ **=>** $f = 42.1\text{ mm}$

[4]

ii) Lens makers formula, $\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2} - \frac{(n-1)d}{n R_1 R_2})$, assume $R_1 = R_2$ and $d \to 0$, $R = 2(n-1)f$ **=>** $R = 58.1\text{ mm}$.

[5]

iii) Set up a Pythagoras relation of lens diameter $h$, and thickness $d$,

$\left(\frac{h}{2}\right)^2 + \left(R - \frac{d}{2}\right)^2 = R^2$, solve for $d$, $d = 2R - 2\sqrt{R^2 - \left(\frac{h}{2}\right)^2}$, for $h = 35\text{ mm}$,

Then $d = 5.40\text{ mm}$

[5]



[END OF PAPER]